n次元におけるチェスのナイトによるハミルトン路問題

Problem of the Hamilton Path with Knight in Chess in n-Dimensions

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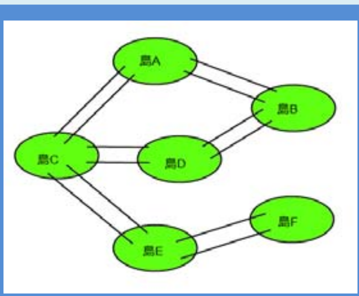
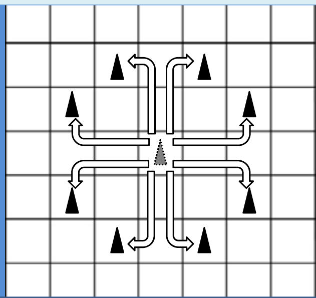


Definition

A Path going along all points of the graph once is called Hamiltonian Path.



The movement of the Knight of the chess is defined to move two blocks to four directions perpendicularly and horizontally, and to move one block moves perpendicularly from there.

Precedent Research

As the end square was different from the starting square, the research we performed was called the Hamiltonian Open Path, but the research of the Hamiltonian Closed Path,which the end point and the starting point is the same, was performed in the name called "Closed Knight 's Tour" in UK. I did not notice until recently.

Knight’s Tours in Higher Dimensions,Joshua Erde,
Electronic Journal of Combinatorics: arXiv:1202.5548v1 [math.CO] 24 Feb 2012

Findings of the Hamiltonian Path by the movement of the Knight in two dimensions

Initial level : Trial and error

7	12	23	18	5
22	17	6	11	24
13	8	15	4	19
16	21	2	25	10
1	14	9	20	3

4	19	22	31	34	7
23	30	5	8	21	32
18	3	20	33	6	35
29	24	27	14	9	12
26	17	2	11	36	15
1	28	25	16	13	10

25	4	15	46	43	6	17
14	41	26	5	16	47	44
3	24	33	42	45	18	7
40	13	38	27	34	31	48
23	2	35	32	37	8	19
12	39	28	21	10	49	30
1	22	11	36	29	20	9

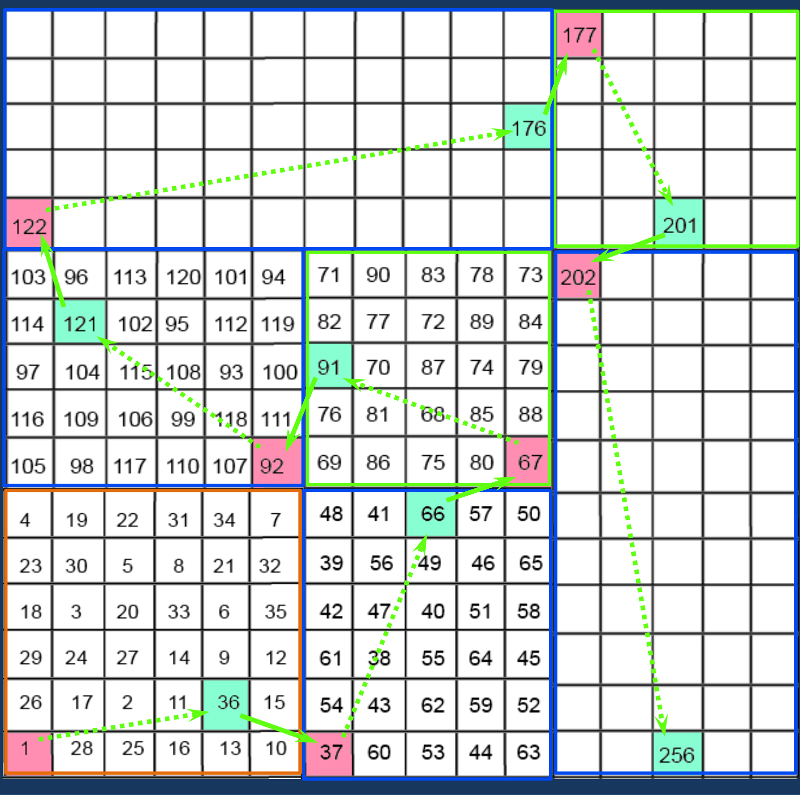
36	57	16	5	34	55	18	7
15	4	35	56	17	6	33	54
58	37	50	39	48	43	8	19
3	14	45	42	51	40	53	32
26	59	38	49	44	47	20	9
13	2	25	46	41	52	31	64
60	27	12	23	62	29	10	21
1	24	61	28	11	22	63	30

I inspected whether squares of n x n had Hamiltonian Path by constituting them concretely. (left-hand figures.) As a result, I confirmed that the Hamiltonian Path existed in the square with block from 5x5 to 20x20. In addition, I showed that there was not Hamiltonian Path in squares of 2x2,3x3nor 4x4.

27	16	9	22	29	14	37	42	47	54	35	62	67	72	79	60
8	23	28	15	10	21	48	53	36	41	46	73	78	61	66	71
17	26	19	4	13	30	43	38	51	34	55	68	63	76	59	80
24	7	2	11	20	5	52	49	32	45	40	77	74	57	70	65
1	18	25	6	3	12	31	44	39	50	33	56	69	64	75	58

Generalization : Idea for the proof

I showed that the Hamiltonian Path which starts in (1,1) , and finished in (n,3) exists in the square of 5 × n(n=5k,5k+1,5k+2,5k+3,5k+4) by connecting rectangle shown in the right diagram.



103	96	113	120	101	94	71	90	83	78	73
114	121	102	95	112	119	82	77	72	89	84
97	104	115	108	93	100	91	70	87	74	79
116	109	106	99	118	111	76	81	68	85	88
105	98	117	110	107	92	69	86	75	80	67
4	19	22	31	34	7	48	41	66	57	50
23	30	5	8	21	32	39	56	49	46	65
18	3	20	33	6	35	42	47	40	51	58
29	24	27	14	9	12	61	38	55	64	45
26	17	2	11	36	15	54	43	62	59	52
1	28	25	16	13	10	37	60	53	44	63

Using this result , I showed that the Hamiltonian Path which starts in (1,1) , and finishes in (n-2,1) exists in the square of n × n.

I guessed n × n is made of (n-5) × (n-5),5 × (n-5), (n-5) × 5 and 5 × 5. Then I put it out for case about remainders when I divided 5 into n and proved it.

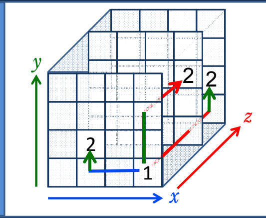
Conclusion 1

In the case of n x n, If n >= 5,there is the Hamiltonian Path that (1,1) is the starting point, and (n-2, 1) becomes the end point.

Findings of the Hamiltonian Path by the Movement of the Knight in the High Dimension

Definition

Movement of the Knight in the n dimensions
I choose an axis to move one block and another axis to move two block from n-axes.
Then I decide which direction to move , plus and minus.



Result of trial and error in 3 and 4 dimensional cases and Conjecture in the n dimension

3dim 4×4×4···4(3)

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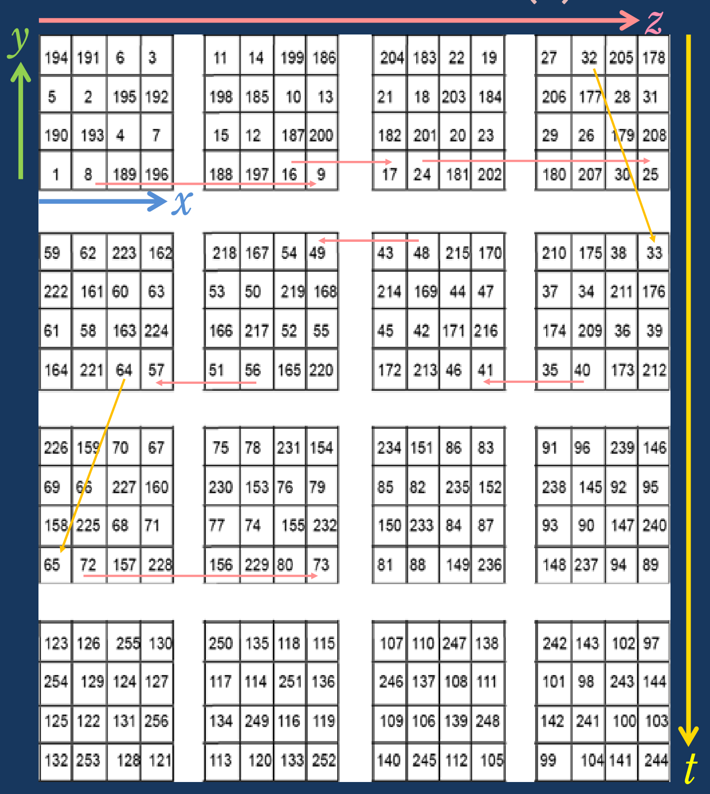

In the two dimensions, as for 4×4, Hamiltonian Path was found not to exist, but, as for 4×4×4 of three dimensions and 4×4×4×4 of the four dimensions, Hamiltonian Path was found to exist.

From this, I conjectured that the condition Hamiltonian Path exists in n×n×・・・×n×n was n >= 4 when the number of dimensions is more than 3.

a(b) refers to case of the b dimension with the a-squares in all directions

Outline of the proof

1. I proof that in the case n≧5,if n(2) with any n has the Hamiltonian Path which starts in (1,1) and ends in (n-2,1), n(m) (m:m >= 2,an integer) also has the Hamiltonian Path.

2. In the case of n ≧ 5, I proof that n(2) has the Hamiltonian path which starts in (1,1) and ends in (n-2,1).

3.In the case of n=4, I show that the Hamiltonian Path which starts in (1,1,1) and finishes in (3,1,4) exists in 4 (3). If it is true,I show that Hamiltonian path exists in 4(m) (m:m >= 3).

Conclusion 2

The condition for n(m) to have the Hamiltonian Path is
n >= 4 and m >= 3 or n >= 5 and m = 2